

# Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <a href="http://about.jstor.org/participate-jstor/individuals/early-journal-content">http://about.jstor.org/participate-jstor/individuals/early-journal-content</a>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

When x=0, (1) becomes

$$\frac{1}{1,2,2,3} - \frac{1}{2,3,3,4} + \frac{1}{3,4,4,5} - \frac{1}{4,5,5,6} + \dots = \frac{\pi^2}{12} - \frac{3}{4}\dots(3).$$

When  $x=\pi$ , (1) becomes

$$\frac{1}{1.2.2.3} + \frac{1}{2.3.3.4} + \frac{1}{3.4.4.5} + \frac{1}{4.5.5.6} + \dots = \frac{7}{4} - \frac{\pi^2}{6} \dots (4).$$

Subtracting (3) from (4), we find the sum of the given series to be  $\frac{5}{4} - \pi^2 / 8$ .

Also solved by J. Scheffer.

315. Proposed by PROFESSOR B. F. YANNEY, Mount Union College, Alliance, Ohio.

Simplify, 
$$1-(2-(3-...-(n-1)-n)...))$$
.

Solution by GEORGE W. HARTWELL, University of Kansas, Lawrence, Kansas, and V. M. SPUNAR, Pittsburg, Pa.

Removing the parentheses, this expression becomes

$$1-2+3-4+\dots(-1)^{n-1}n \equiv \sum_{1}^{n} (-1)^{n-1}n.$$

But  $\sum_{1}^{n} (-1)^{n-1}n = -(n/2)$  when *n* is even,

and 
$$\sum_{1}^{n} (-1)^{n-1} n = (n+1)/2$$
 when *n* is odd.

Also solved by G. B. M. Zerr.

#### GEOMETRY.

342. Proposed by G. I. HOPKINS, M. A., Instructor in Mathematics and Astronomy, Manchester, N. H.

Given, circle DEF inscribed in triangle ABC and circumscribing the triangle DEF, D, E, F being the points of contact; AH is drawn through center, N, meeting chord DF in H. Through H is drawn BK meeting AC in K. Prove triangle ABK isosceles.

I. Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Let the points D, F, E be situated on the sides a, b, c, respectively, and also let  $l=\cos^2(A/2)$ ,  $m=\cos^2(B/2)$ ,  $n=\cos^2(C/2)$ . Then (0, rn, rm); (rn, 0, rl), are the trilinear coordinates of D and F, respectively.

Hence  $\beta - \gamma = 0$  is the equation to AN,  $l + m \beta - n \gamma = 0$  is the equation to DF.

 $\therefore$  (n-m, l, l);  $(0, 2 \triangle /b, 0)$ , are the coordinates of H and B, respectively.  $\therefore l + (m-n)\gamma = 0$  is the equation to BK.

But  $m-n+l\cos C+(m-n)\cos A-l\cos B=0$ .

 $\therefore BK$  is perpendicular to AN.  $\therefore$  triangle ABK is isosceles.

### II. Solution by G. I. HOPKINS, Instructor in Mathematics and Astronomy, High School, Manchester, N. H.

Construction: Join HE, HB. Extend DE and draw BP perpendicular to it.

Demonstration: Since BP and AH are perpendicular to DE, they are parallel. AD=AE, i. e.,  $\triangle ADE$  is isosceles. arc ES=arc SF.

- $\therefore \angle ENB = \angle EDF$ .  $\therefore \triangle DRH$  and  $\triangle NEB$  are similar.
- $\therefore NE/EB = DR/RH$ .  $\angle NEB$  is a right angle.
- $\therefore \angle REN$  is complement to  $\angle BEP$ .  $\therefore \angle REN = \angle EBP$ .
- $\therefore \triangle REN$  and  $\triangle EBP$  are similar.
- $\therefore RE/BP = NE/EB; \therefore DR/RH = RE/BP.$

Since DR=RE,  $\therefore RH=BP$ , and  $\therefore RHBP$  is a parallelogram; i, e., BK is parallel to DE.

 $\therefore \triangle ABK$  is similar to  $\triangle ADE$ , and is therefore isosceles.

## CALCULUS.

270. Proposed by S. A. COREY, Hiteman, Iowa.

Prove that  $\sum_{x=0}^{x=\infty} \frac{1}{(a^2+x^2)^n} = \frac{\pi}{2a^{2n-1}} \cdot \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \cdot ... \cdot \frac{(2n-3)}{(2n-2)} + \frac{1}{2a^{2n}}$ , n being a positive integer >1.

#### II. Solution by the PROPOSER.

Performing the finite summation of the problem by the aid of Maclaurin's Summation-formula,

$$\Sigma u_x = C + \int u_x dx - \frac{1}{2}u_x + \frac{B_1}{2!} \frac{du_x}{dx} - \frac{B_2}{4!} \frac{d^3 u_x}{dx^3} + \dots$$

(See Boole's *Finite Differences*, page 90), we readily obtain the above expression for the sum, if we substitute for the definite integral,  $\int_0^\infty \frac{dx}{(a^2+x^2)^n}$ , its well known value,  $\frac{\pi}{2} \cdot \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \cdot \cdot \cdot \frac{(2n-3)}{(2n-2)} \cdot \frac{1}{a^{2n-1}}$ , if n is a positive integer >1.

The solution in the May Monthly involves the erroneous assumption that  $\sum_{x=0}^{x=\infty} \frac{1}{(a^2+x^2)^n} = \int_0^{\infty} \frac{dx}{(a^2+x^2)^n}$ .